tone phase, lb. solute/lb. nonsolute
Z = effective column height, in.
ρ_i = density of liquid i, lb./cu. ft.

Subscripts

c = continuous phase
d = dispersed phase
i = indicates either phase can be used throughout the equation where i appears
k = ketone phase
w = water phase
1 = bottom of the column
2 = top of the column

Superscript

$*$ = equilibrium concentration

LITERATURE CITED

1. Ballard, John H., and Edgar L. Piret, *Ind. Eng. Chem.*, **42**, 1088 (1950).
2. Belaga, M. W., and J. E. Bigelow, *U.S. Atomic Energy Commission Report KT-133* (1952).
3. Buchanan, R. H., *Aust. J. Appl. Sci.*, **3**, 233 (1952).
4. Burkhart, L. E., and R. W. Fahien, *U.S. Atomic Energy Commission Report ISC-860* (1956).
5. Burns, W. A., and W. F. Johnson, *U.S. Patent 2 662 001/1953.*
6. Cervi, A. F., Thesis, Columbia University, New York (1943).
7. Choffe, B., and Y. L. Gladel, *J. Appl. Chem.*, **8**, 580 (1958).
8. Cohen, R. M., and G. H. Beyer, *Chem. Eng. Progr.*, **49**, 279 (1953).
9. Garner, F. H., S. R. M. Ellis, and D. W. Fosbury, *Trans. Inst. Chem. Engrs. (London)*, **31**, 348 (1953).
10. Garner, F. H., S. R. M. Ellis, and J. W. Hill, *A.I.Ch.E. Journal*, **1**, 185 (1955).
11. ———, *Trans. Inst. Chem. Engrs. (London)*, **34**, 223 (1956).
12. Johnson, H. F., and Harding Bliss, *Trans. Am. Inst. Chem. Engrs.*, **42**, 331 (1946).
13. Licht, William, and J. B. Conway, *Ind. Eng. Chem.*, **42**, 1151 (1950).
14. Livingston, M. J., *U.S. Patent 2 215 359/1940.*
15. Mayfield, F. D., and W. L. Church, *Ind. Eng. Chem.*, **44**, 2253 (1952).
16. Oldshue, J. Y., and J. H. Rushton, *Chem. Eng Progr.*, **48**, 297 (1952).

16a. Othmer, D. F., R. E. White, and E. Treuger, *Ind. Eng. Chem.*, **33**, 1240 (1941).

17. Pike, F. P., *U.S. Atomic Energy Commission Report ORNL-929*, (1951).

17a. Scheibel, E. G., and A. E. Karr, *Ind. Eng. Chem.*, **42**, 1048 (1950).

18. Sege, G., and F. W. Woodfield, *Chem. Eng. Progr.*, **50**, 396 (1954).
19. Sherwood, T. K., J. E. Evans, and J. A. Longcor, *Ind. Eng. Chem.*, **31**, 1144 (1939).
20. Sherwood, T. K., and R. L. Pigford, "Absorption and Extraction," McGraw-Hill, New York (1952).

20a. Sobotik, R. H., M. S. thesis, Univ. Tex., Austin, Texas (1959).

21. Treybal, R. E., *Ind. Eng. Chem.*, **34**, 709 (1942).
22. ———, "Mass-Transfer Operations," McGraw-Hill, New York (1955).
23. West, F. B., P. A. Robinson, A. C. Morgenthaler, Jr., T. R. Beck, and D. K. McGregor, *Ind. Eng. Chem.*, **43**, 234 (1950).

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Effects of Solids on Turbulence in a Fluid

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The effect of solid particles on fluid turbulence was studied for fully developed flows of slurries in a vertical 3-in. pipe for solids concentrations ranging from 0.13 to 2.5 volume %. Point source turbulent diffusion data in the slurry flows were compared with data for flows without solids present. The solids do not appear to have a large effect on the diffusion rate unless there is an appreciable average slip velocity between the solids and the fluid and unless the solids concentration is high enough.

A theoretical treatment of turbulent flow involving the suspension of solids in a fluid would use information on turbulent flow of single-phase systems as a starting point. The question arises whether the presence of the solids affects the fluid turbulence. Very little information is available in the literature on the interaction of solid particles with a turbulent field; therefore this study was undertaken to gain some insight regarding the conditions under which the presence of solid particles would change the fluid turbulence. In particular the effect of solids on the rate of diffusion in a fluid was studied by comparing data on point source turbulent diffusion with and without solids in the field. The technique is similar to that used in a previous study of turbulent diffusion in fluidized beds (*4*).

Water and solid-water suspensions were circulated through a 3-in. vertical pipe. A 1N solution of potassium chloride was injected through a small tube in the center of the pipe at a sufficient distance from the pipe inlet such that the flow was fully developed. Samples were withdrawn from the flow over a period of time, long compared with the time scale of the turbulence. The samples were taken across diameters at several distances from the plane of the injector. The spread of the material at a plane downstream from the injector could be measured by the mean square of the x-component of the displacement of the diffusing particles, $\overline{X^2}$. The variation of $\overline{X^2}$ with distance from the injector is a measure of the rate of diffusion. The variation of $\overline{X^2}$ at large times measures the gross properties of the turbulent mixing process and is influenced by the largest eddies. The effect of the presence of solid particles on such measurements reflects the interaction of the particles with the large scale fluid eddies.

During the course of this investigation Soo (*11*) reported results on a similar study. He carried out measurement in a horizontal pipe on the diffusion of helium gas in air and in suspensions which contained 0.0005 to 0.0030 volume % glass spheres of diameter less than 0.01 in. He found that the presence of the solids did not affect the diffusion in the fluid phase. The volume percent and the type of solids which Soo could investigate were limited by fall out of the particles from the flow stream. By carrying out experiments in a vertical pipe larger solids concentrations and larger particle densities can be investigated. For flow in a vertical pipe a region far from the entry

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is reached in which the concentration of particles does not vary with height in the pipe. In this region the average velocity of the particles could be the same or less than the velocity of the fluid. The difference between the average velocity of the particles and the average velocity of the fluid is called the *slip velocity*. Although the particles are falling with reference to the fluid, they still have a net velocity in the direction of flow and are transported out of the pipe. For horizontal flows the average fluid velocity in the direction of slip is zero. Therefore particles with appreciable slip velocity cannot be supported in a horizontal pipe flow.

The results of this investigation indicate that only the solids which have a large free-fall velocity in the stagnant fluid appreciably affect the large-scale turbulence responsible for diffusion. For a suspension flowing turbulently in a pipe the slip velocity might be different from the free-fall velocity. However some recent experiments by Wolfe and Murphy (*13*) indicate that for the systems studied in this research the free-fall velocity is probably a close approximation to the slip velocity. Direct measurements of particle velocities by a photographic technique were made for the flow of very dilute suspensions in a vertical 1.5-in. pipe over a range of particle Reynolds numbers of 7.5 to 127 and Reynolds numbers based on the tube diameter of 5,600 to 32,200. Slip velocities calculated from these measurements agree within about 10% of the settling velocity in nonturbulent systems. An exact mathematical treatment of the effect of turbulence on the slip velocity does not exist. A discussion of the problem will be presented below for the case of dilute suspension in a homogeneous flow field. Some ways by which the slip velocity could affect turbulent diffusion in the fluid will also be discussed.

EFFECT OF TURBULENCE ON THE SLIP VELOCITY FOR A HOMOGENEOUS FLOW FIELD

An exact expression may be written for the equation of motion of a particle in a fluid for the case in which the relative velocity between the solid and the fluid is small (*8*). An exact expression for the general case cannot be written because the effect of acceleration on the drag coefficient cannot be defined. On the basis of a discussion presented by Hughes and Gilliland (*5*) the equation of motion for a small solid spherical particle in a fluid may be written as follows:

$$\rho_s \frac{4}{3}\pi a^3 \frac{d\vec{V}}{dt} = \frac{4}{3}\pi a^3 (\rho_s - \rho) \vec{g} - \frac{c_d}{2}\rho \pi a^2 |w|^2 \frac{\vec{w}}{|w|} - \frac{4}{3}\pi a^3 \overrightarrow{\text{grad}\, p} - m\frac{4}{3}\pi a^3 \rho \frac{d\vec{w}}{dt} \quad (1)$$

The left side of the equation is the product of the particle mass and the particle acceleration. The first term on the right side of the equation is the force of gravity on the particle minus the buoyancy effect of the fluid. The second term is the resistance of the fluid to the motion of the particle. The third term is the force on the particle due to pressure gradients in the fluid other than those due to hydrostatic head. The fourth term is an assumed virtual mass effect which represents the acceleration of the particle with reference to the fluid. For a perfect fluid the coefficient m is given as one half (*12*). It can be shown that m also is one half for a viscous fluid in which the relative motion between the particle and fluid is small (*8*). It will be assumed that the effect of acceleration of the particle will be taken into account in the term containing the virtual mass and therefore that the drag coefficient c_d will be a function only of a particle Reynolds number equal to $2a\rho|w|/\mu$.

If the flow is upward and if one considers only the particle motion in the direction of the tube axis, Equation (1) takes the following form:

$$\rho_s \frac{4}{3}\pi a^3 \frac{dV_z}{dt} = -\frac{4}{3}\pi a^3 (\rho_s - \rho) g - \frac{c_d}{2}\rho \pi a^2 |w|\, w_z - \frac{4}{3}\pi a^3 \frac{\partial p}{\partial z} - m\rho\frac{4}{3}\pi a^3 \frac{dw_z}{dt} \quad (2)$$

Equation (2) will be applied to a suspension dilute enough that the particles behave independently of one another and at a distance far enough from the entry that the flow is fully developed. The flow field will be assumed homogeneous. The velocities of the particles and the fluid they contact may be defined in terms of an average and fluctuating component: $V_z = \overline{V_z} + V_z'$, and $u_z = \overline{u_z} + u_z'$. If these expressions are substituted into Equation (3) and the entire equation is averaged, the following equation results:

$$\rho_s \frac{4}{3}\pi a^3 \overline{\frac{dV_z}{dt}} = -\frac{4}{3}\pi a^3(\rho_s - \rho) g - \frac{1}{2}\rho\pi\, a^2 \overline{c_d\, |w|\, (V_z - u_z)} - \frac{4}{3}\pi a^3 \overline{\frac{\partial p}{\partial z}} - m\rho\frac{4}{3}\pi a^3 \overline{\frac{dw_z}{dt}} \quad (3)$$

Since the time derivatives in the above equation are taken along the particle path

$$\overline{\frac{dV_z}{dt}} = \overline{\frac{\partial V_z}{\partial t}} + \overline{V_x \frac{\partial V_z}{\partial x}} + \overline{V_y \frac{\partial V_z}{\partial y}} + \overline{V_z \frac{\partial V_z}{\partial z}} \quad (4)$$

$$\overline{\frac{dw_z}{dt}} = \overline{\frac{\partial w_z}{\partial t}} + \overline{V_x \frac{\partial w_z}{\partial x}} + \overline{V_y \frac{\partial w_z}{\partial y}} + \overline{V_z \frac{\partial w_z}{\partial z}} \quad (5)$$

For a homogeneous flow field and for a fully developed flow of a suspension the above two terms are zero. Likewise for the experiments reported in this paper $\overline{\partial p/\partial z}$ may be neglected, and Equation (3) reduces to the following expression:

$$0 = -\frac{4}{3}\pi a^3 (\rho_s - \rho) g - \frac{\pi a^2 \rho}{2} \overline{c_d |w| w_z} \quad (6)$$

For cases in which Stoke's law is applicable

$$c_d = 24.0 \Big/ \frac{2 a\rho |w|}{\mu} \quad (7)$$

and Equation (6) becomes

$$-\overline{w_z} = \frac{2}{9}\frac{a^2}{\mu}(\rho_s - \rho) g \quad (8)$$

If a slip velocity w_s is defined as

$$w_s = \overline{V_z} - \overline{u_z} \quad (9)$$

then for the Stoke's law region

$$-w_s = \frac{2}{9}\frac{a^2}{\mu}(\rho_s - \rho) g \quad (10)$$

This result shows that for a homogeneous flow the slip velocity is not affected by turbulence as long as Stoke's law is applicable. However for slip velocities large enough such that Stoke's law is not applicable the effect of the turbulence is reflected in the term $\overline{c_d |w| w_z}$. No expression which relates this term to measurable turbulence quantities is evident to the authors. However if c_d and $|w|$ are defined as

$$c_d = \frac{K}{|w|^n} \quad (11)$$

$$|w| = [(V_z - u_z)^2 + (V_x - u_x)^2 + (V_y - u_y)^2]^{1/2} \quad (12)$$

the following equation results:

$$\frac{8}{3} a \left(\frac{\rho_s - \rho}{\rho}\right) g \left[1 + \frac{V'_z - u'_z}{w_s}\right] = \frac{- K\, w_s |w_s|^{1-n}}{\left[1 + 2\left(\frac{V'_z-u'_z}{w_s}\right) + \left(\frac{V'_z-u'_z}{w_s}\right)^2 + \left(\frac{V'_y-u'_y}{w_s}\right)^2 + \left(\frac{V'_x-u'_x}{w_s}\right)^2\right]^{\frac{1-n}{2}}} \quad (13)$$

The correction for the effect of turbulence is contained in the terms in the brackets. If the term in the brackets is expanded in terms of $(V'_z - u'_z)$, $(V'_y - u'_y)$, and $(V'_x - u'_x)$, before averaging

$$\frac{8}{3} a \left(\frac{\rho_s - \rho}{\rho}\right) g = - K\, w_s\, |w_s|^{1-n}$$

$$(1-n)\,\frac{1}{2}\left[\overline{\left(\frac{V'_z - u'_z}{w_s}\right)^2} + \overline{\left(\frac{V'_y - u'_y}{w_s}\right)^2} + \overline{\left(\frac{V'_x - u'_x}{w_s}\right)^2}\right] + \frac{(1-n)^2}{2}\,\overline{\left(\frac{V'_z - u'_z}{w_s}\right)^2}$$

+ terms involving higher order correlations (14)

It can be seen that the effect of turbulence depends on the slip arising from the fluctuations in the fluid and particle velocities. The magnitude of these correction terms will depend on the magnitude of the turbulence intensity and on the lag of the particles in following fluctuations in the velocity of the fluid. For a homogeneous isotropic fluid (3, 11) the terms $\overline{\left(\frac{V'_z - u'_z}{w_s}\right)^2}$, $\overline{\left(\frac{V'_y - u'_y}{w_s}\right)^2}$, and $\overline{\left(\frac{V'_x - u'_x}{w_s}\right)^2}$ will vary between zero and $\overline{u'^2_z}$. For a nonhomogeneous field in which there exists gradients in the velocity it is difficult to put any limits on these quantities.

In all of the results presented in this paper the slip velocity w_s has been calculated on the assumption that turbulence has no effect. Values of the slip velocities and the particle Reynolds numbers calculated from these slip velocities are tabulated in Table I. In all cases the particle Reynolds number is larger than that which would be required for Stoke's law to be valid. It is likely therefore that the calculated slip velocities are subject to some error. In order to use Equation (14) to obtain an estimate of the magnitude of the error due to neglecting the effect of turbulence it was assumed that the magnitudes of $\overline{\left(\frac{V'_z - u'_z}{w_s}\right)^2}$, $\overline{\left(\frac{V'_x - u'_x}{w_s}\right)^2}$, and $\overline{\left(\frac{V'_y - u'_y}{w_s}\right)^2}$ are small enough such that higher-order correlations may be neglected and that these mean square slip velocities are of the magnitude of the turbulent intensity. With turbulence measurements of Sandborn used, slip velocities calculated from Equation (14) for the large glass spheres and for the copper shot are tabulated in Table I. These calculations indicate that the turbulence would exert a small influence on the slip velocity.

ENERGY INPUT TO THE FLUID DUE TO PARTICLE SLIP

If there is an average slip between the particles and the fluid, mechanical energy is being fed to the fluid by the force of gravity acting upon the particles. This energy input can be calculated for the fully developed flow of a suspension by applying the laws of conservation of momentum and of energy to the section of the flow field shown in Figure 1. The law of conservation of momentum states that the net rate of flow of momentum out of the control volume is equal to the force acting on it. Since a fully developed flow is being considered, the net flow of momentum out is zero and the force will result from the force of gravity, the pressure drop, and the drag at the wall:

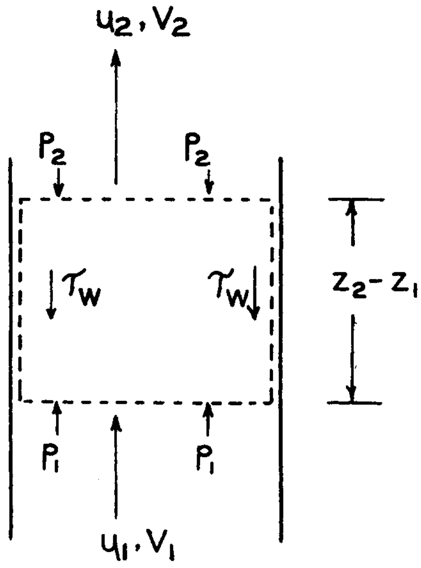

Fig. 1. Control volume in the fully developed flow region.

$$\text{Net force} = 0 \quad (15)$$

$$(p_1 - p_2)\, S - S\,(Z_2 - Z_1)\, c\, \rho_s\, g - S\,(Z_2 - Z_1)(1-c)\rho\, g - \tau_w\, \pi D\,(Z_2 - Z_1) = 0 \quad (16)$$

$$(p_1 - p_2) = c\,(Z_2 - Z_1)\, g\,(\rho_s - \rho) + (Z_2 - Z_1)\,\rho g + \tau_w \frac{\pi D}{S}(Z_2 - Z_1) \quad (17)$$

If a quantity U is defined as the internal energy of the slurry mixture per unit mass of mixture, an energy balance may be written for the volume shown in Figure 1. This energy balance will yield an expression for the rate of dissipation of energy per pound of fluid, signified by ϵ and having the

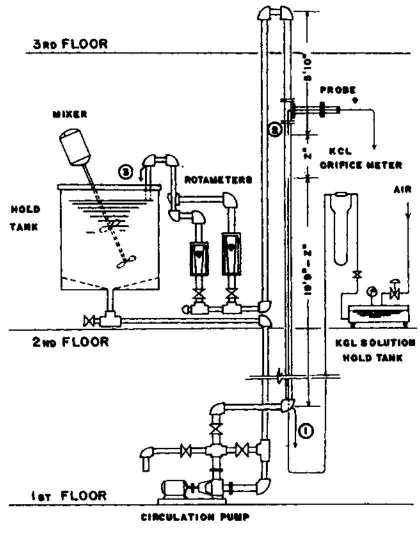

Fig. 2. Drawing of equipment.

TABLE 1. CALCULATED SLIP VELOCITIES

Solids	D_P, (in.)	$\rho_s - \rho$, (g./cc.)	$\lvert w_s\rvert$*, (in./sec.)	$\frac{D_P \lvert w_s\rvert \rho}{\mu}$	$\lvert w_s\rvert$ [Equation (14)] $N_{Re} = 20{,}300$, (in./sec.)	$\lvert w_s\rvert$ [Equation (14)] $N_{Re} = 50{,}550$, (in./sec.)
Glass	0.00394	1.20	0.26	0.66		
Glass	0.01488	1.20	1.91	18.3	1.85	1.5
Copper	0.00788	7.92	3.55	18.1	3.5	3.4

* Free-fall velocity of the solid particle in a static fluid.

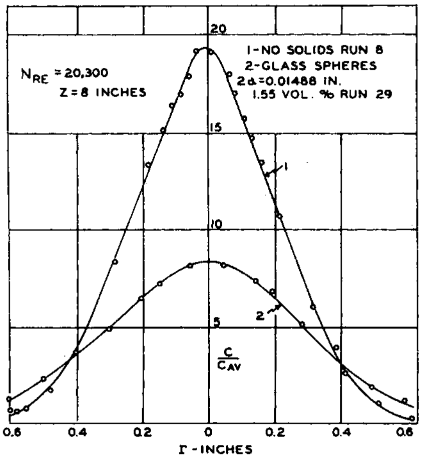

Fig. 3. Concentration profile demonstrating large effect of solids on diffusion data.

units square feet divided by seconds cubed. The law of conservation of energy states that the net flow of energy out of the volume will equal the rate of heat flow through the walls minus the rate at which the fluid in the volume is doing work on the surroundings. The energy terms included in the balance are an internal energy, a potential energy due to the movement through the gravitational field, and the kinetic energies of the solids and the fluid.

If the flow is assumed uniform and if W_s and W are the weight flow of solids and of fluid

$$(W_s + W)(U_2 - U_1) + (W_s + W) g(Z_2 - Z_1) + W_s\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) + W\left(\frac{u_2^2}{2} - \frac{u_1^2}{2}\right) = q\,\pi D (Z_2 - Z_1) - \frac{W_s}{\rho_s}(p_2 - p_1)$$

$$- \frac{W}{\rho}(p_2 - p_1) \qquad (18)$$

The rate at which mechanical energy is dissipated per unit mass of fluid may be defined as follows:

$$\epsilon = \frac{(W_s + W)(U_2 - U_1)}{S(1-c)\,\rho\,(Z_2 - Z_1)} - \frac{q\,D(Z_2 - Z_1)}{S(1-c)\,\rho\,(Z_2 - Z_1)} \qquad (19)$$

Using Equation (18) and the assumption of a fully developed flow one gets

$$\epsilon = -\frac{(W_s + W)g}{S(1-c)\rho} - \frac{W_s(p_2 - p_1)}{\rho_s S(1-c)\rho(Z_2 - Z_1)} - \frac{W(p_2 - p_1)}{\rho^2 S(1-c)(Z_2 - Z_1)} \qquad (20)$$

From the definition of the slip velocity

$$\overline{V_z} = \overline{u_z} + w_s$$

$$W_s = (\overline{u_z} + w_s)\, S c \rho_s \qquad (21)$$

$$W = \rho \overline{u_z}\, S(1-c) \qquad (22)$$

$$W_s = \left[\frac{W}{\rho S(1-c)} + w_s\right] S c\, \rho_s \qquad (23)$$

If the above expressions for W_s and W are substituted into Equation (20), and if Equation (17) is substituted for $-(p_2 - p_1)$, the following expression is obtained for ϵ:

$$\epsilon = -c\left(\frac{\rho_s - \rho}{\rho}\right) g\, w_s + \frac{\left[1 + c\frac{w_s}{\overline{u_z}}\right] 4\,\tau_W \overline{u_z}}{(1-c)\rho D} \qquad (24)$$

The first term gives the energy input to the fluid due to the fall out of the solids. The solids act as mixers, and the force of gravity does work on the system by pulling the solids through the fluid at an average velocity w_s relative to the fluid. The second term is the energy input into the fluid by the pump to overcome the resisting force at the wall.

For $c = 0$, an expression for the energy input to the fluid for the case of no solids is obtained:

$$\epsilon_0 = \frac{4\,\tau_W \overline{u_z}}{\rho D} \qquad (25)$$

The Fanning friction coefficient is defined as

$$f = \frac{\tau_w}{1/2\,\rho\,\overline{u_z}^2} \qquad (26)$$

When one uses this expression, the energy dissipation for flow without solids is

$$\epsilon_0 = \frac{2f\,\overline{u_z}^3}{D} \qquad (27)$$

EFFECT OF SOLIDS SLIP ON POINT-SOURCE DIFFUSION

The presence of solids in a turbulently flowing fluid can affect the rate of diffusion in the fluid by changing the character of the fluid turbulence or by the mechanical mixing resulting from the relative motion of the solids and the fluid. The solids should affect the fluid turbulence if they are larger than the smallest eddies or if there is a relative motion between the solids and the fluid. This relative motion can result from the velocity fluctuations of the solids and fluid being out of phase. The quantities $\overline{(V'_z - u'_z)^2}$, $\overline{(V'_y - u'_y)^2}$, and $\overline{(V'_x - u'_x)^2}$ would be measures

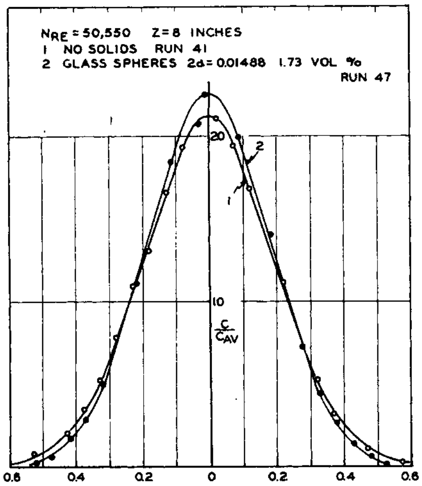

Fig. 4. Concentration profile demonstrating negligible effect of solids on diffusion data.

TABLE 2. SUMMARY OF RESULTS

System	Solids volume %	Re	$\frac{w_s}{u_c}$	$\frac{\epsilon_s}{\epsilon_0}$	E, (in.²/sec.)	β	Remarks
No particles		20,300			0.0397		
Glass	1.5	20,300	0.021	0.41	0.0407	—	Negligible effect
$D_P = 0.00394$ in.	(1.2-1.6)						
	2.5	20,300	0.021	0.68	0.0515	49	Moderate effect
Glass	0.5	20,300	0.15	1.00	0.0363	—	Negligible effect
$D_P = 0.01488$ in.	(0.25-0.82)						
	1.5	20,300	0.15	3.00	0.1020	28	Large effect
	(1.3-2.2)						
	2.5	20,300	0.15	5.00	0.1103	23	Large effect
	(1.9-3.2)						
Copper	0.13	20,300	0.29	3.20	0.0490	37	Moderate effect
$D_P = 0.00788$ in.							
No particles		50,550			0.0744		
Glass	1.7	50,550	0.062	0.24	0.0744	—	Negligible effect
$D_P = 0.01488$ in.							
	2.3	50,550	0.062	0.40	0.0833	8	Moderate effect

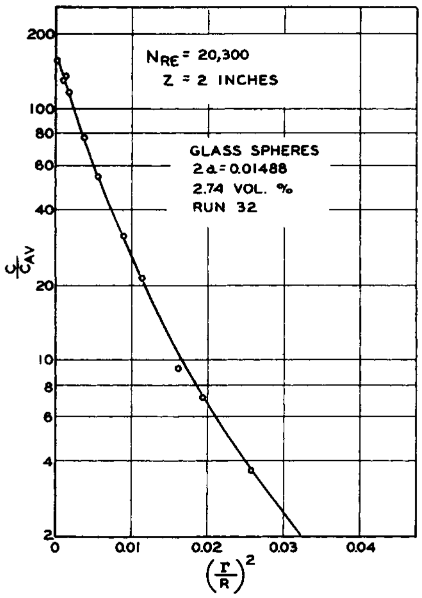

Fig. 5. Non-Gaussian profile indicated by non-linear plot of log C/C_A vs. $\left(\frac{r}{R}\right)^2$.

of this effect. It can also be due to an average slip velocity resulting from the force of gravity acting upon the solids. This slip velocity results in an additional energy input to the fluid over that which would be dissipated by the fluid in the absence of the solids. Some of this additional energy might first be used to increase the intensity of the turbulence before being dissipated into heat. A measure of the magnitude of this effect would be the ratio of the energy input due to the gravitational pull on the solids to the energy input in the absence of solids. From Equations (24) and (27) this ratio is

$$\frac{\epsilon_s}{\epsilon_0} = \frac{-c\,\dfrac{\rho_s - \rho}{\rho}\, g\, w_s\, D}{2f\,\bar{u}_z^{\,3}} \qquad (28)$$

The presence of the solids could also change the energy input by affecting

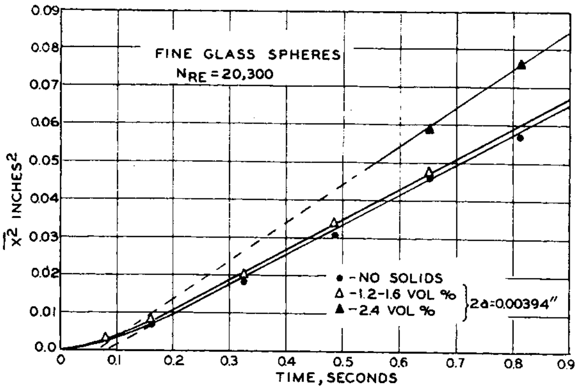

Fig. 7. Effect of solids on $\overline{x^2}$ vs. t data.

the wall shear stress. In cases in which there is appreciable slip velocity this effect will probably be small compared with the increase of energy input due to solids fall out.

The mechanical mixing due to the presence of solids results from the displacement of fluid masses caused by the relative velocity between the solids and the fluid and by the rate of change of this relative velocity. For cases in which w_s is large the mechanical mixing might be described in the same manner as in a fixed bed (*1*, *7*). Assume that, on an average, the fluid in passing a particle undergoes a displacement which is some multiple of the particle radius

$$\overline{x^2} = \beta^2 a^2 \qquad (29)$$

The particles in falling through the fluid with a slip velocity will on an average sweep out a volume of fluid

$$Q_s = n_s\, |w_s|\, \pi a^2 t \qquad (30)$$

The volumetric throughput of fluid is

$$Q = \bar{u}_z\, \pi R^2\, (1 - c)\, t \qquad (31)$$

If it is assumed that the number of encounters which a particle of fluid has with solid particles is proportional to the ratio of Q_s/Q

$$N = K\frac{n_s\, |w_s|\, a^2}{\bar{u}_z (1 - c) R^2} \qquad (32)$$

Now n_s may be related to the volume per cent of solids:

$$\frac{4}{3}\pi a^3 n_s = c\,\pi R^2 z$$

By eliminating n_s from Equation (32) one obtains

$$N = K\frac{3}{4}\frac{|w_s|}{\bar{u}_z}\frac{c}{(1 - c)}\frac{z}{a} \qquad (33)$$

The total fluid displacement due to all solid particle encounters is

$$\overline{X^2} = N\,\overline{x^2} = K\beta^2\frac{3}{4}\frac{|w_s|}{\bar{u}_z}\frac{c}{(1 - c)}a^2\frac{z}{a} \qquad (34)$$

A diffusion coefficient may be defined as

$$E = \frac{\overline{X^2}}{2t} = \frac{\overline{X^2}\,\bar{u}_z}{2z} \qquad (35)$$

Assuming $K = 1$, one gets

$$E = \frac{3}{8}\beta^2\, |w_s|\, a\,\frac{c}{1 - c} \qquad (36)$$

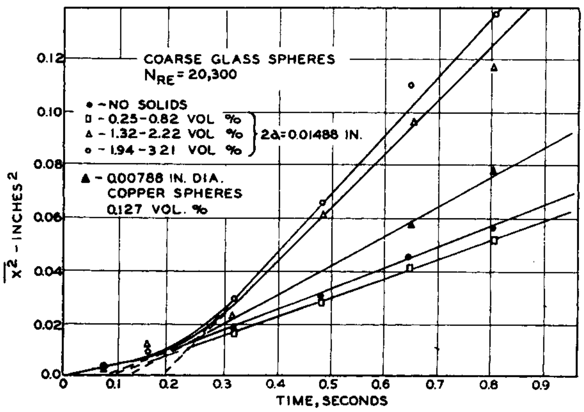

Fig. 6. Effect of solids on $\overline{x^2}$ vs. t data.

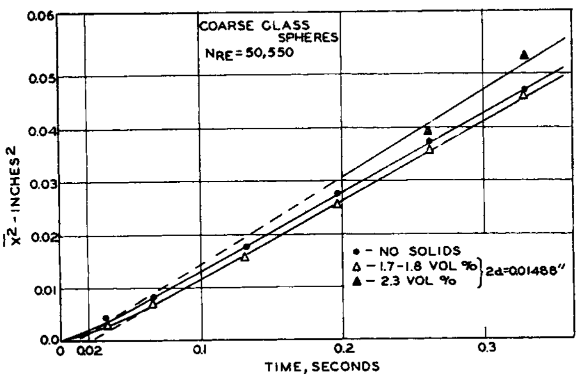

Fig. 8. Effect of solids on $\overline{x^2}$ vs. t data.

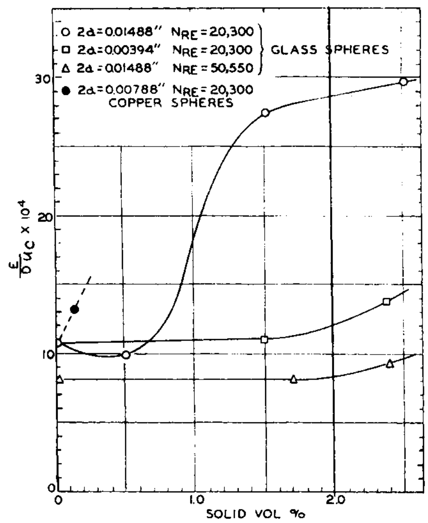

Fig. 9. Effect of solids concentration on the turbulent diffusion coefficient.

For a packed bed the coefficient β has a value of about 0.6. Owing to the sideways motion of the particles it would be expected that β could be larger for the turbulent flow of a suspension.

EXPERIMENTAL

The mass diffusion experiments were conducted in a 3-in. I.D. Pyrex pipe through which dimineralized water or a slurry of solids in demineralized water was circulated. The solids were glass spheres of average diameter 0.01488 and 0.00394 in., and a few runs were made with copper spheres of average diameter 0.00788 in. The concentration of solids ranged from 0.5 to 2.5 volume %. The specific gravity of the glass particles was 2.20, and the specific gravity of the copper particles was 8.92. One normal potassium chloride solution was injected into the water through a small tube which was centrally located in the pipe. Samples were removed from the flow stream at different locations along a diameter by a stainless steel tube which was driven by a micrometer screw. A settling chamber was attached to the sampling probe to catch solid particles before collecting the sample. The distance between the plane of the injector and the plane from which samples were withdrawn was varied by changing the injector position. The water samples were analyzed for potassium chloride content by measuring their electroconductivity. A schematic drawing of the apparatus is shown in Figure 2. The glass pipe was 25 ft. long to allow for the full development of the turbulent flow field. The test section was 19 to 20 ft. from the inlet to the column. A detailed description of the equipment and the experimental techniques is presented in another article (*2*) and in a thesis by one of the authors (*6*).

TREATMENT OF DATA

All of the concentration profiles except those obtained with systems in which the solids were affecting the diffusion appreciably were Gaussian. For Gaussian profiles

$$C\, 2\pi\, r dr dz = \frac{Q dt}{\overline{X^2}} e^{-\frac{r^2}{2\overline{X^2}}} dr \tag{37}$$

Substituting

$$u_c = \frac{dz}{dt} \tag{38}$$

and

$$Q = \pi R^2 u_{AV} C_{AV} \tag{39}$$

one obtains the following equation, from which $\overline{X^2}$ can be calculated:

$$\frac{C}{C_{AV}} = \frac{R^2 u_{AV}}{2 u_c \overline{X^2}} e^{-\frac{r^2}{2\overline{X^2}}} \tag{40}$$

For non-Gaussian profiles $\overline{X^2}$ was obtained by graphical integration:

$$\overline{X^2} = \frac{\overline{R^2}}{2} = \frac{1}{2} \frac{\int_0^R 2\pi r^3 C dr}{\int_0^R 2\pi r\, C dr} \tag{41}$$

Values of $\overline{X^2}$ calculated from measured profiles were plotted vs. time, defined as

$$t = \frac{z}{u_c} \tag{42}$$

A turbulent diffusion coefficient was defined from the limiting slope of the $\overline{X^2}$ vs. t curve:

$$E = \frac{1}{2}\left(\frac{d\overline{X^2}}{dt}\right)_{t\to\infty} \tag{43}$$

RESULTS

Measurements were made at Reynolds numbers of 20,300 and 50,550 with slurries of the particles described in Table 1 over a concentration range of 0.13 to 2.5 volume %. The effect of the solids upon mass diffusion for the different systems investigated is characterized as large, moderate, or negligible in Table 2. The solids concentrations shown in Table 2 are averages. The actual concentrations ranged over the values indicated inside the parentheses. A comparison of concentration profiles of the diffusing potassium chloride solution in a slurry with empty tube data is shown in Figure 3 for a run in which a large effect was noted and in Figure 4 for a run in which a negligible effect was noted. The concentration profiles were Gaussian error curves in all cases except those in which large effects were noted. In these experiments a flaring of the profiles as shown in Figure 5 was obtained. The magnitude of this flaring decreased with increasing distance from the plane of the injector. Mean-squared displacements $\overline{X^2}$ calculated from measured concentration profiles are shown in Figures 6, 7, and 8. A marked difference from the data with no solids is obtained for the runs with 0.01488-in. glass spheres at solids concentrations of 1.5 and 2.5 volume % Values of E were calculated from the slopes of the $\overline{X^2}$ vs. t curves at large values of t. These are presented in Table 2 and are plotted in Figure 9. Only two profiles were used to calculate E for the copper shot and for the runs at a Reynolds number of 50,550 with 2.5 volume % of glass spheres.

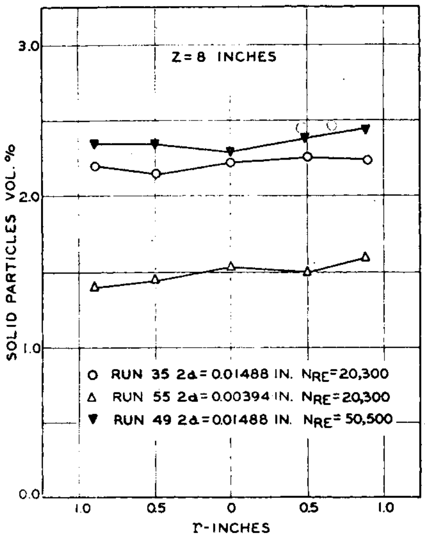

Fig. 10. Distribution of solid particles.

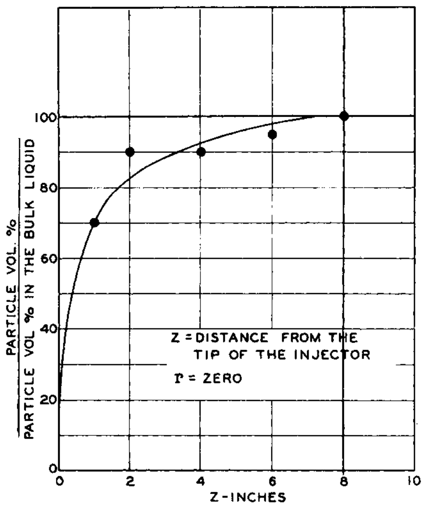

Fig. 11. Solid particle concentration profile on the axis of the injector.

The concentration of solids in the sampling stream was measured for a number of runs with the glass spheres. Some of these data are presented in Figures 10 and 11. The data indicated that the solids were uniformly dispersed in the pipe. However, as shown in Figure 11, the solids concentration was low right above the tip of the injector.

DISCUSSION

If it can be assumed that the turbulence is not appreciably affecting the slip velocity, then the results of this research indicate that the effect of the solids on the rate of diffusion in the fluid depends on the solids concentration and on the ratio of the average slip velocity to the fluid velocity. Thus 1.5% of glass spheres for which $w_s/u_c = 0.15$ produces a 2.5-fold increase in the turbulent diffusion, while a 1.5% slurry for which $w_s/u_c = 0.021$ gives approximately the same diffusion coefficient as a flow without solids. Likewise only 0.13% of copper particles for which $w_s/u_c = 0.29$ gives approximately the same effect as 2.5% of glass spheres for which $w_s/u_c = 0.021$. The energy input to the fluid by gravity acting upon the solids was calculated for the systems studied in this research, and the results of these calculations are presented in Table 2. For the cases in which the solids had a large effect on the mass diffusion the energy input to the fluid was 3 to 5 times as large as the energy dissipated by the fluid in absence of solids.

An increase in the Reynolds number decreases the effect of the solids on the fluid turbulence. At $Re = 20{,}300$, a 2.5- to 2.8-fold increase in the diffusion coefficient was obtained for 1.5 to 2.5 volume % of 0.01488-in. glass spheres. At $Re = 50{,}550$, the diffusion coefficient was 1 to 1.1 times the diffusion coefficient without spheres for the same solids concentrations. This effect can be explained in terms of the decrease of w_s/u_c and ϵ_s/ϵ_0.

Two aspects of these results are surprising. First it is possible to have concentrations of solids as high as 1.5% without greatly affecting the fluid turbulence responsible for turbulent diffusion. The second is the sudden increase in eddy diffusion coefficient illustrated in Figure 9. This suggests a marked change in the properties of the system. This was somewhat borne out by visual observations. It was noted that the character of the solids motion changed as the concentration increased. At low concentrations the solids moved independently of one another and had very small velocity fluctuations in the radial direction. Their motion was essentially linear. As the concentration increased groups of particles tended to move in unison, the particle velocities in the radial direction increased, and there were large fluctuations in the concentration of the solids.

Values of β defined by Equation (36) on the assumption that $K = 1$ have been calculated from the difference of the measured diffusion coefficient and the diffusion coefficient for an empty tube. The magnitudes of the calculated β's seem too large for the displacement mechanism of turbulent diffusion described in Equation (36) to account for all of the observed increase in the diffusion rate.

The non-Gaussian concentration profiles obtained in some runs might be explained by the smaller concentration of solids in the vicinity of the injector. For runs in which the solids were greatly influencing the diffusion rate the material would spread slower directly over the injector than at positions farther away.

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NOTATION

- a = radius of a particle
- c = volume fraction of solids
- c_d = drag coefficient
- C = concentration of diffusing material
- C_{AV} = mixed average concentration
- D = diameter of the pipe
- D_P = particle diameter
- E = eddy diffusion coefficient
- f = Fanning friction factor
- g = acceleration of gravity
- K = proportionality constant
- m = constant used to describe the magnitude of the virtual mass
- n_s = number of particles
- N = number of encounters
- p = pressure
- q = heat flow per unit area of pipe wall
- Q_s = volume of fluid swept out
- r = radial position
- R = radius of the pipe
- S = cross-sectional area of the pipe
- t = time of diffusion, equal to $z/\overline{u_z}$
- u = fluid velocity
- u' = fluctuating fluid velocity
- u_z = fluid velocity in the z-direction
- u_c = fluid velocity at the pipe center
- u_{AV} = mixed average fluid velocity
- U = internal energy per unit weight of suspension
- V = particle velocity
- V' = fluctuating particle velocity
- V_z = particle velocity in the z-direction
- W = weight flow of fluid
- W_s = weight flow of solids
- $\vec{w}$ = relative velocity between the particle and the fluid $= \overline{V} - \vec{u}$
- w_s = slip velocity $= \overline{V_z} - \overline{u_z}$
- $\overline{x^2}$ = effective displacement per particle encounter
- $\overline{X^2}$ = x-component of the mean squared displacement of the diffusing material
- $|\ |$ = brackets signify absolute value
- z = distance between the plane of the injector and the plane of sampling

Greek Letters

- β = coefficient defined by Equation (26)
- ϵ = rate of energy dissipation per unit mass of fluid
- ϵ_s = energy dissipation due to the average slippage of the particles
- ϵ_0 = energy dissipation for a flow without solids
- μ = viscosity
- ρ = fluid density
- ρ_s = particle density
- τ_w = wall shear stress

LITERATURE CITED

1. Baron, Thomas, *Chem. Eng. Progr.*, 48, 118 (1952).
2. Flint, Duane L., Hisao Kada, and Thomas J. Hanratty, *A.I.Ch.E. Journal*, 6, No. 2, p. 325 (1960).
3. Friedlander, S. K., *ibid.*, 3, 381 (1957).
4. Hanratty, Thomas J., G. A. Latinen, and R. H. Wilhelm, *ibid.*, 2, 372 (1956).
5. Hughes, R. R., and E. R. Gilliland, *Chem. Eng. Progr.*, 48, 497 (1952).
6. Kada, Hisao, Ph.D. thesis, Univ. of Illinois, Urbana, Illinois (March, 1959).
7. Latinen, G. A., Ph.D. thesis, Princeton Univ., Princeton, New Jersey (December, 1951).
8. Lumley, John L., Ph.D. thesis, The Johns Hopkins University, Baltimore, Maryland (May, 1957).
9. Perry, John H., "Chemical Engineers' Handbook," 3 ed., McGraw-Hill, New York (1950).
10. Sandborn, V. A., *Natl. Advisory Comm. Aeronaut. Tech. Note 3266* (1955).
11. Soo, S. L., H. K. Ihrig, Jr., and A. F. El Kouh, Project Squid, *Technical Report PR-82-P*, Princeton University, Princeton, New Jersey (September, 1958).
12. Streeter, Victor L., "Fluid Dynamics," 1 ed., p. 64, McGraw-Hill, New York (1948).
13. Wolfe, H. E., and Glenn Murphy, *U.S.A.E.C. Report ISC-878*, Ames Laboratory, Iowa State College, Ames, Iowa (March, 1957).

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